Quantum neural networks

Reference: ECB, J.E. Steck, P. Kumar, & K.A. Walsh, "Quantum algorithm design using dynamic learning," QIC **8**, 0012-0029 (2008).
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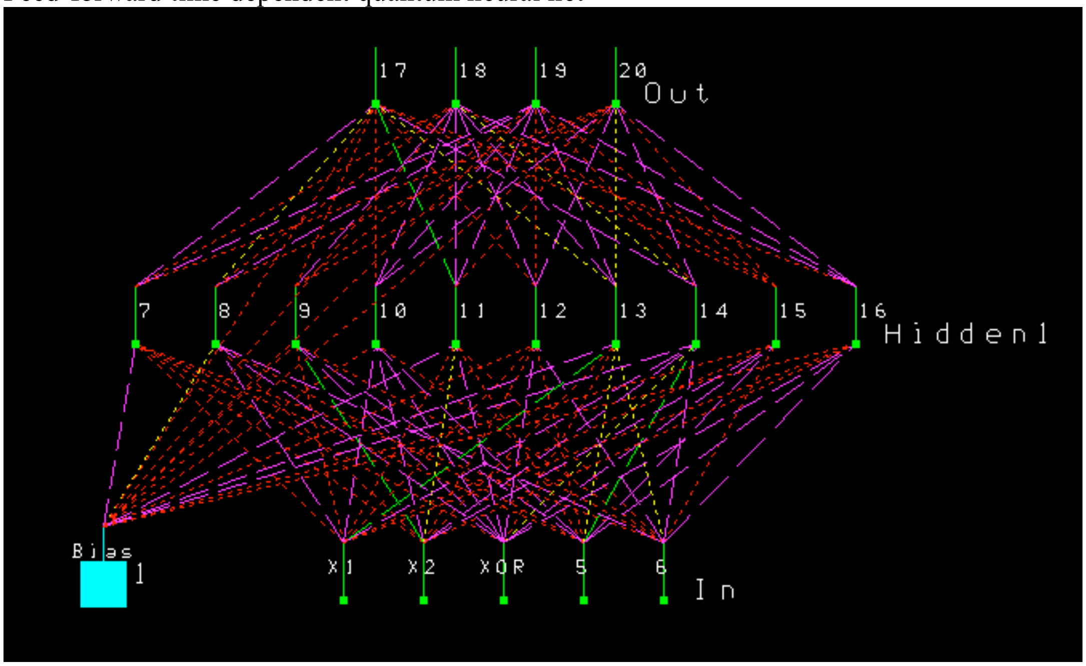
Feed-forward time dependent quantum neural net



Physical model: SQuID qubits. For two qubits this is

$$H = K_A\sigma_{XA} + K_B\sigma_{XB} + \varepsilon_A\sigma_{ZA} + \varepsilon_B\sigma_{ZB} + \xi_{AB}\sigma_{ZA}\sigma_{ZB}$$

- Given an initial state, ρ(0), the system evolves in time according to the Schrödinger equation: $$\frac{\partial\rho}{\partial t} = -\frac{i}{\hbar}[H,\rho]$$
- We construct a Lagrangian to be minimized: $$L = \frac{1}{2}\left[d - \langle M(T)\rangle\right] + \int_0^T \lambda^+(t)\left(\frac{\partial\rho}{\partial t} + \frac{i}{\hbar}[H,\rho]\right)\gamma(t)dt,$$
- Where we define the output measure M by:

$$\begin{aligned} Out &= \langle M(T)\rangle = tr(\rho(T)M) \\ &= \sum_i p_i|\psi_i(T)\rangle\langle\psi_i(T)|M \\ &= \sum_i p_i\langle\psi_i(T)|M|\psi_i(T)\rangle, \end{aligned}$$

We take the first variation of L with respect to ρ, set it equal to zero, then integrate by parts:

$$\lambda_i\frac{\partial\gamma_j}{\partial t} + \frac{\partial\lambda_i}{\partial t}\gamma_j - \frac{i}{\hbar}\sum_k \lambda_k H_{ki}\gamma_j + \frac{i}{\hbar}\sum_k \lambda_i H_{jk}\gamma_k = 0,$$

with the boundary conditions at the final time T given by

$$-\left[d - \langle M(T)\rangle\right]M_{ji} + \lambda_i(T)\gamma_j(T) = 0\cdot$$

we update the weights according to

$$w_{new} = w_{old} - \eta \frac{\partial L}{\partial w},$$

$$\frac{\partial L}{\partial w} = \frac{i}{\hbar}\int_0^T \lambda^+(t)\left[\frac{\partial H}{\partial w}, \rho\right]\gamma(t)dt$$

$$= \frac{i}{\hbar}\int_0^T \sum_{i,j,k}\left(\lambda_i(t)\frac{\partial H_{ik}}{\partial w}\rho_{kj}\gamma_j - \lambda_i(t)\rho_{ik}\frac{\partial H_{kj}}{\partial w}\gamma_j\right)dt$$

and repeat. Each repetition through the training set is called an "epoch".

Logic gates:

Input	XNOR		XOR	
	Target	Output	Target	Output
$\|\uparrow\uparrow\rangle$	-1	-0.9919	+1	0.9902
$\|\uparrow\downarrow\rangle$	+1	0.9920	-1	-0.9903
$\|\downarrow\uparrow\rangle$	+1	0.9902	-1	-0.9919
$\|\downarrow\downarrow\rangle$	-1	-0.9903	+1	0.9920
	rms = 0.00446; epochs = 300		rms = 0.00447; epochs = 300	

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Initial state	Target	Trained	Targets		Trained	
$\|\uparrow\uparrow\uparrow\rangle$	+1	0.9828	+1	+1	0.9910	0.9873
$\|\uparrow\uparrow\downarrow\rangle$	-1	-0.9828	+1	-1	0.9958	-0.9960
$\|\uparrow\downarrow\uparrow\rangle$	+1	0.9928	-1	+1	-0.9998	0.9995
$\|\uparrow\downarrow\downarrow\rangle$	-1	-0.9929	-1	-1	-0.9870	-0.9908
$\|\downarrow\uparrow\uparrow\rangle$	+1	0.9935	+1	+1	0.9785	0.9787
$\|\downarrow\uparrow\downarrow\rangle$	-1	-0.9936	-1	+1	-0.9944	0.9943
$\|\downarrow\downarrow\uparrow\rangle$	-1	-0.9960	+1	-1	0.9945	-0.9944
$\|\downarrow\downarrow\downarrow\rangle$	+1	0.9960	-1	-1	-0.9786	-0.9786
	rms=0.00356; epochs=300		rms=0.00279; epochs=1500			

<table>
<tr><th>(unnormalized) initial state</th><th>Target</th><th>Output</th></tr>
<tr><td>|↑↑> +|↓↓></td><td>1.0</td><td>0.99997</td></tr>
<tr><td>|↑↑> +|↑↓> +|↓↑> +|↓↓></td><td>0.0</td><td>2.01×10^{-6}</td></tr>
<tr><td>|↑↑> +0.5| ↑↓></td><td>0.0</td><td>2.61×10^{-5}</td></tr>
<tr><td>|↑↓> +|↓↑> +|↓↓></td><td>0.44317</td><td>0.44317</td></tr>
</table>

Entanglement training

and testing

<table>
<tr><th>State</th><th>Target</th><th>Output</th></tr>
<tr><td>|↑↓> ±|↓↑></td><td>1.0</td><td>1.00000</td></tr>
<tr><td>{(α|↑>+β|↓>)$_A$ (γ|↑>+δ|↓>)$_B$}</td><td>0.0</td><td>rms error{10000} = 1.1×10^{-7}</td></tr>
<tr><td>{α |↑↑>< ↑↑|+ β|↓↑><↓↑| + γ|↑↓>< ↑↓|+ δ|↓↓><↓↓|}</td><td>0.0</td><td>rms error{10000} = 2.6×10^{-7}</td></tr>
<tr><td>|↑↑> +|↓↑> +|↓↓></td><td>0.44317</td><td>0.44317</td></tr>
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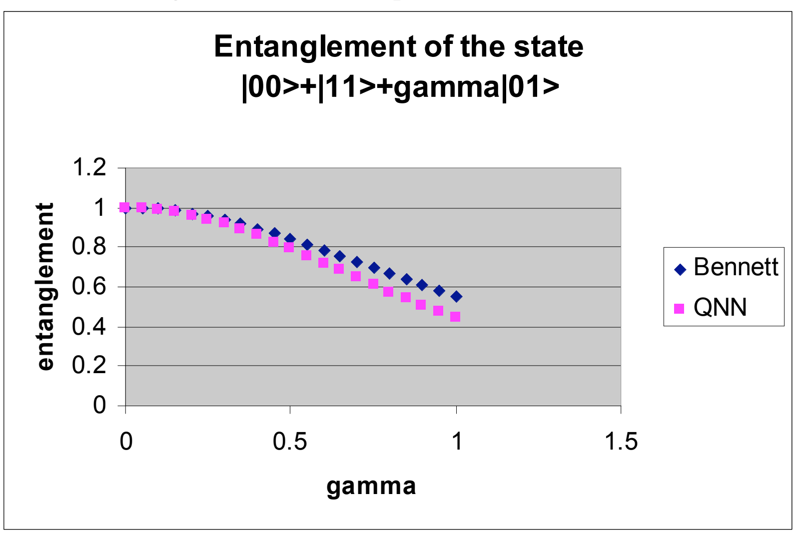
pure entanglement: comparison with Bennett-Wootters entropy of

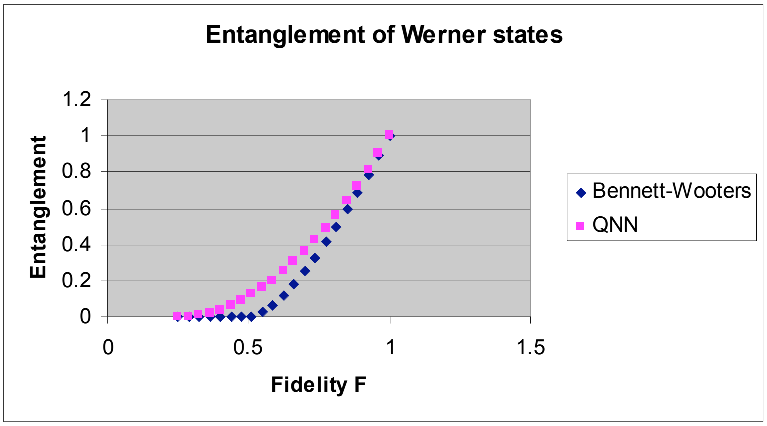


formation

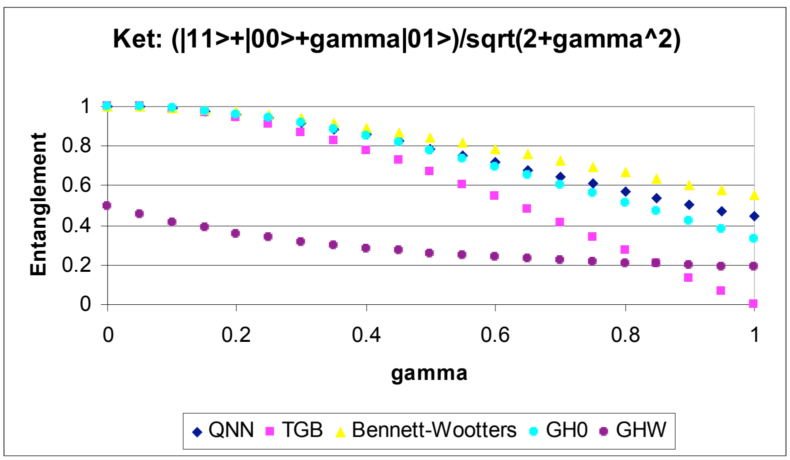
Partially entangled mixed states:

Werner state $W=F|\psi^-\rangle\langle\psi^-| + ((1-F)/3)(|\psi^+\rangle\langle\psi^+| + |\Phi^+\rangle\langle\Phi^+| + |\Phi^-\rangle\langle\Phi^-|)$

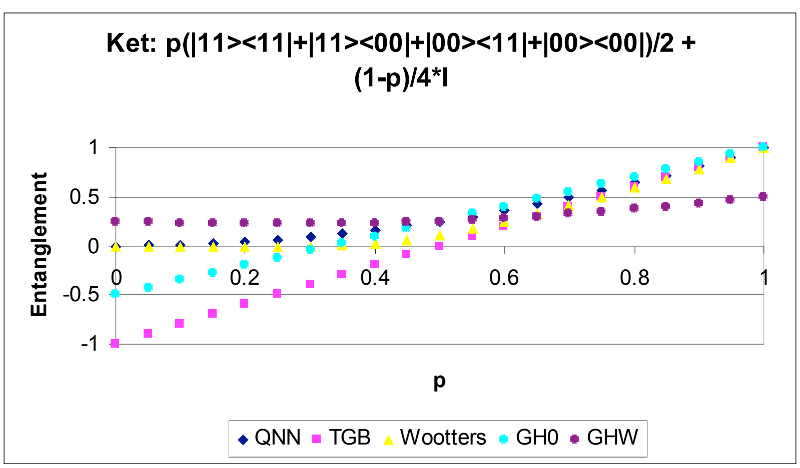
$x=(4F-1)/3$ parts pure singlet (fully entangled); $(1-x)$ parts identity operator



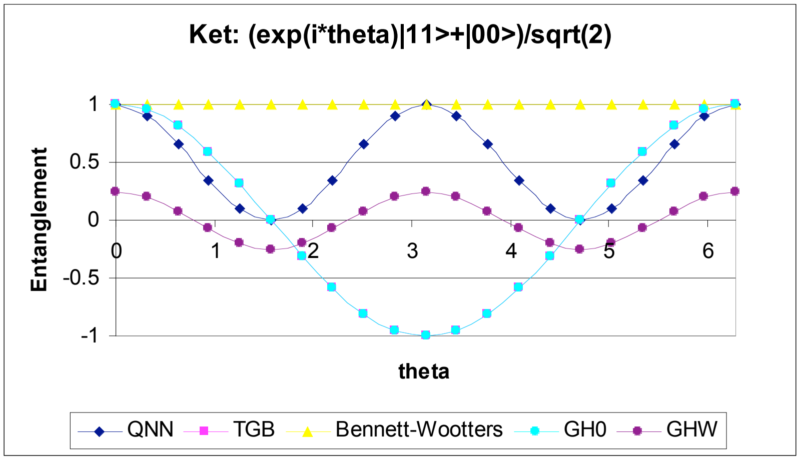
comparison with witnesses



Effect of noise:



relative phase problem.



What else?

- Entanglement typing?
- Error correction? - can do detection of flippage, correction of flippage; both together cheaplier
- Degree of W-state entanglement?: To distinguish among:
  |0>|0>(|01>+|10>), |0>(|001>+|010>+|100>),
  |0001>+|0010>+|0100>+|1000> - Experimentally accessible! (HJ Kimble, Science 324, 764 (2009))